

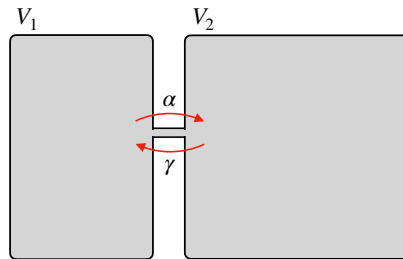
Stochastic Processes

Final exam, 08/02/2022
09:15 — 12:30

Problem 1:

Two containers of volumes V_1 and V_2 filled with a gas are in thermal equilibrium. The total number of gas particles is N . The containers are connected by a narrow opening allowing for a slow exchange of particles.

- Write down the master equation for the probability p_n that n particles are in the left container. Assume that processes in which particles are moving from one container to the other are independent from each other and the jump probabilities per unit time for a single particle are α and γ , as indicated in the figure.
- Find the stationary solution for this situation.
- Find the relation between α and γ .



Problem 2:

Consider a Rayleigh particle in an environment at temperature T . Its velocity is described by the Langevin equation

$$m\dot{v} = -\zeta v + L(t),$$

where $L(t)$ is a Gaussian white noise. Initially, at $t = 0$ the particle is located at $x = 0$ and has the velocity v_0 . Find the probability distribution for the particle's position $P(x, t; v_0)$ for this situation.