Stokes flow generated by a point force in various geometries
I. Pressure field

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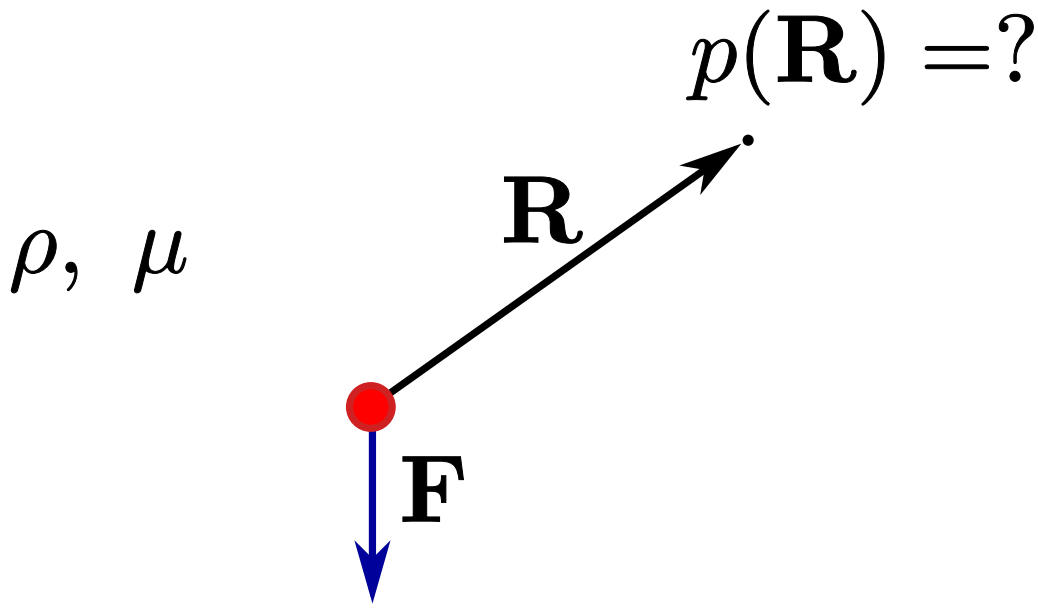
Pressure field



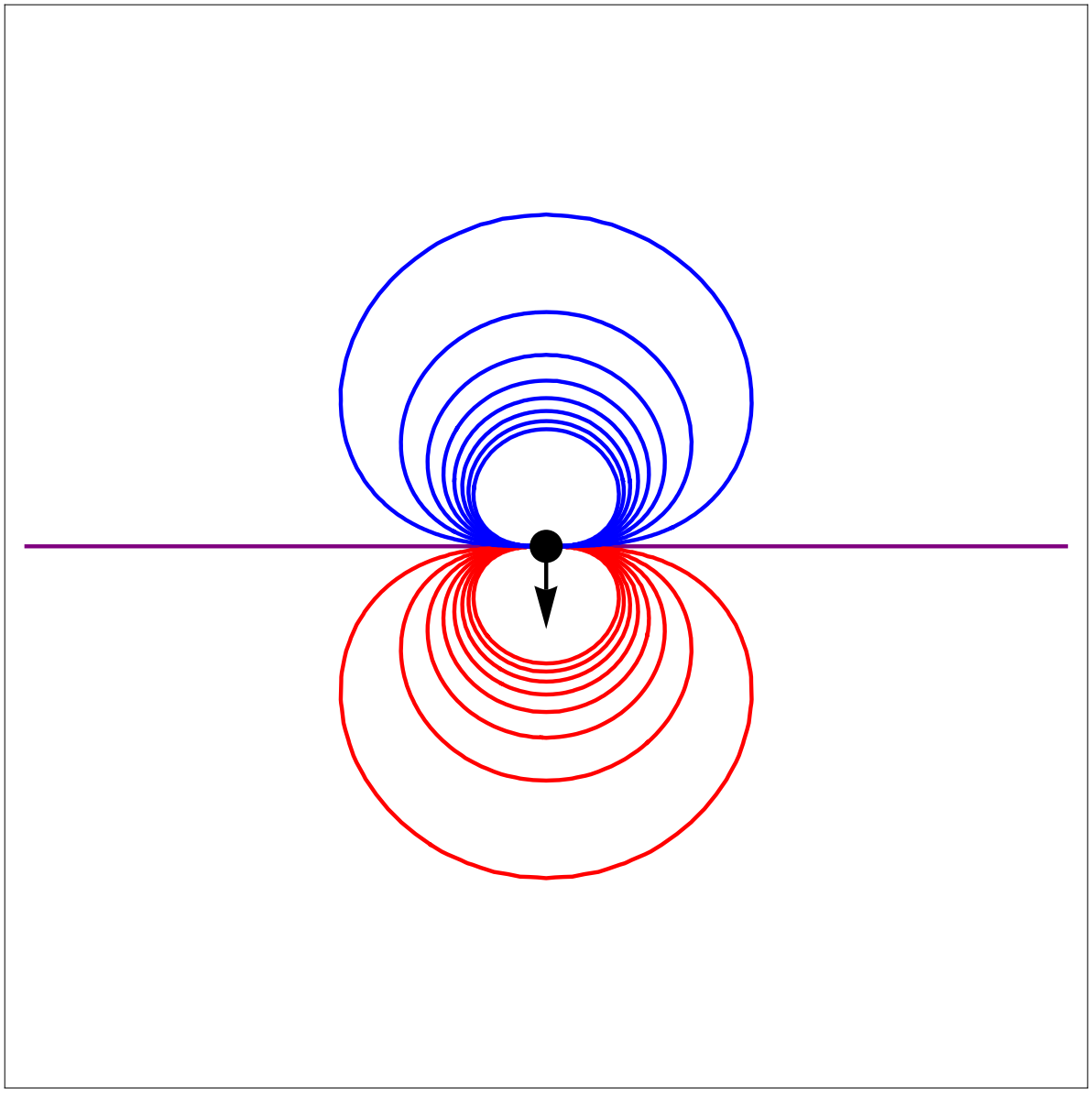
How do the isobars look like?

Geometry

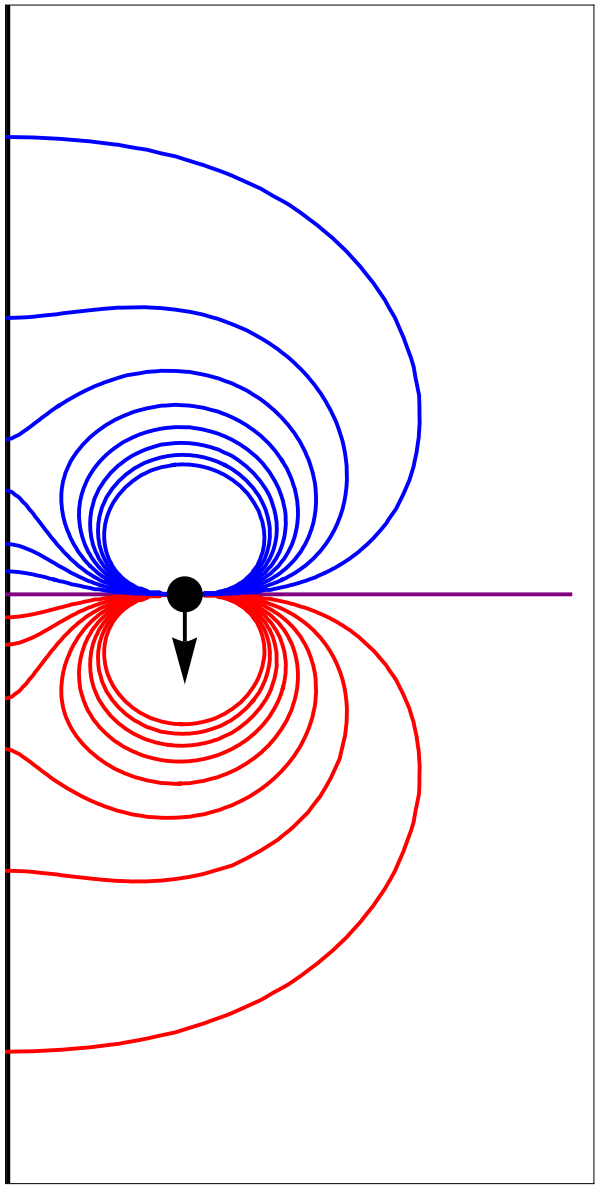
Point force in a viscous fluid:

- Unbounded
- With a free boundary
  - Parallel to the force
  - Perpendicular to the force
- With a rigid wall
  - Parallel to the force
  - Perpendicular to the force

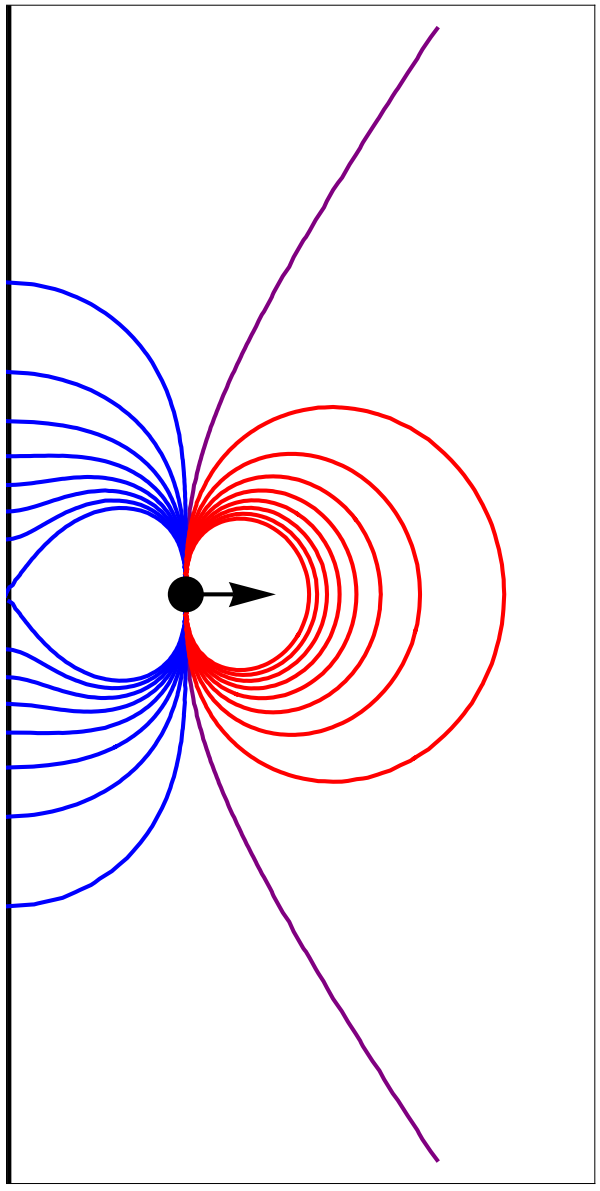
Unbounded fluid



Free surface (thick black line on the left)

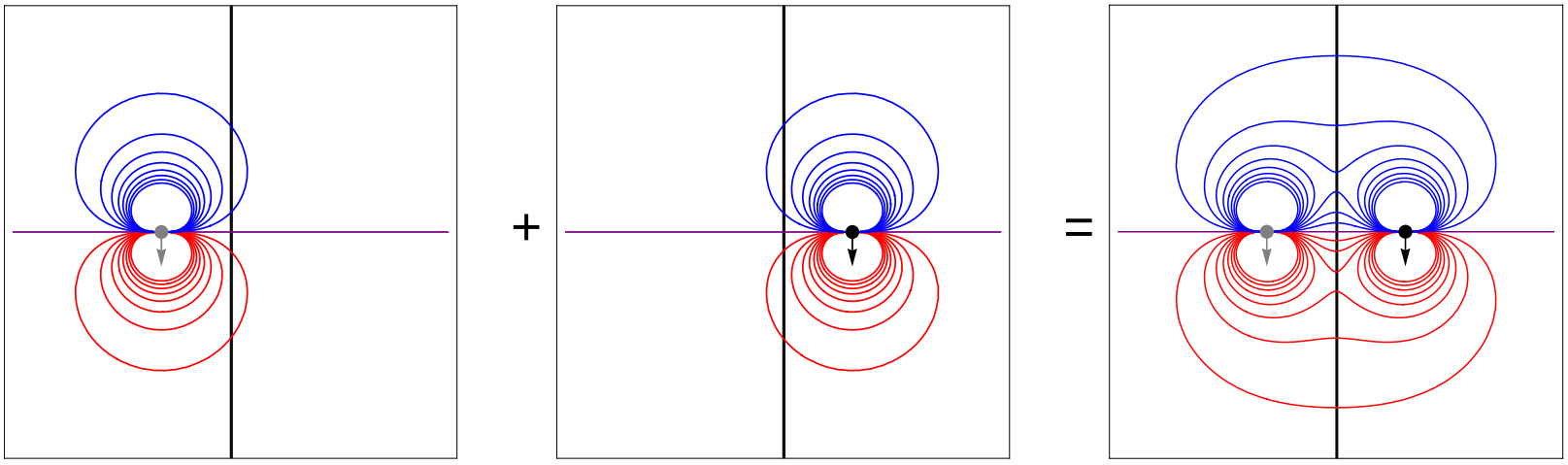


Parallel force

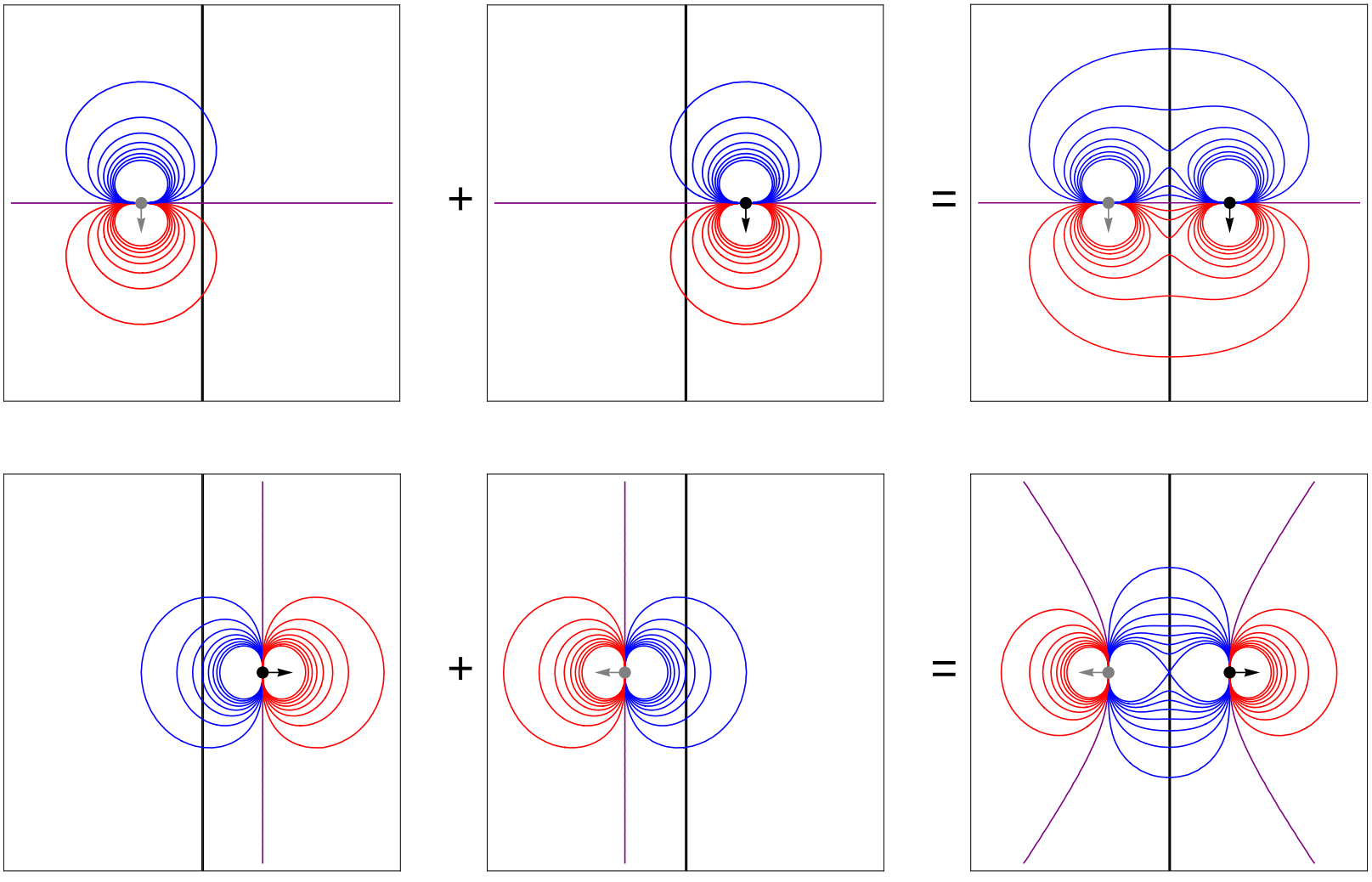


Perpendicular force

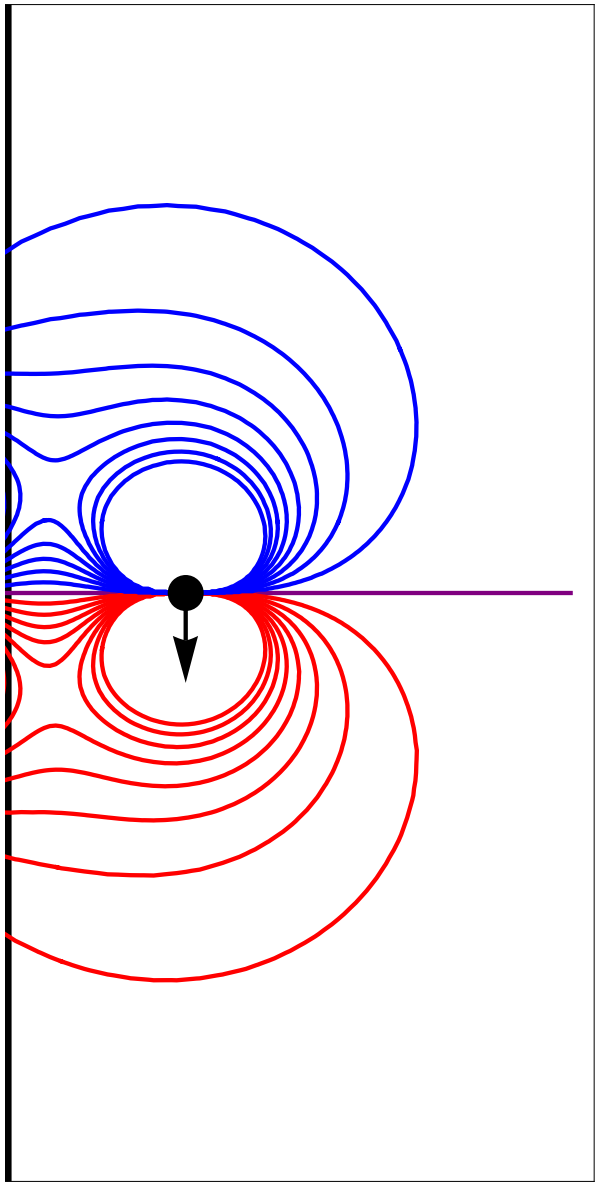
Free surface - the method of images



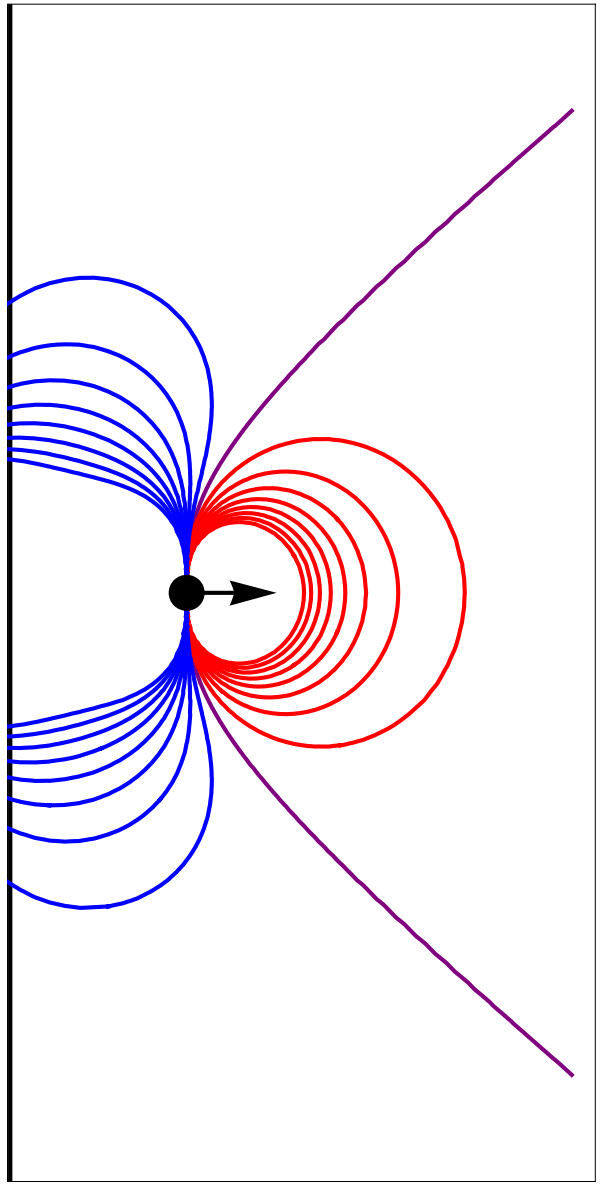
Free surface - the method of images



Rigid wall (thick black line on the left)

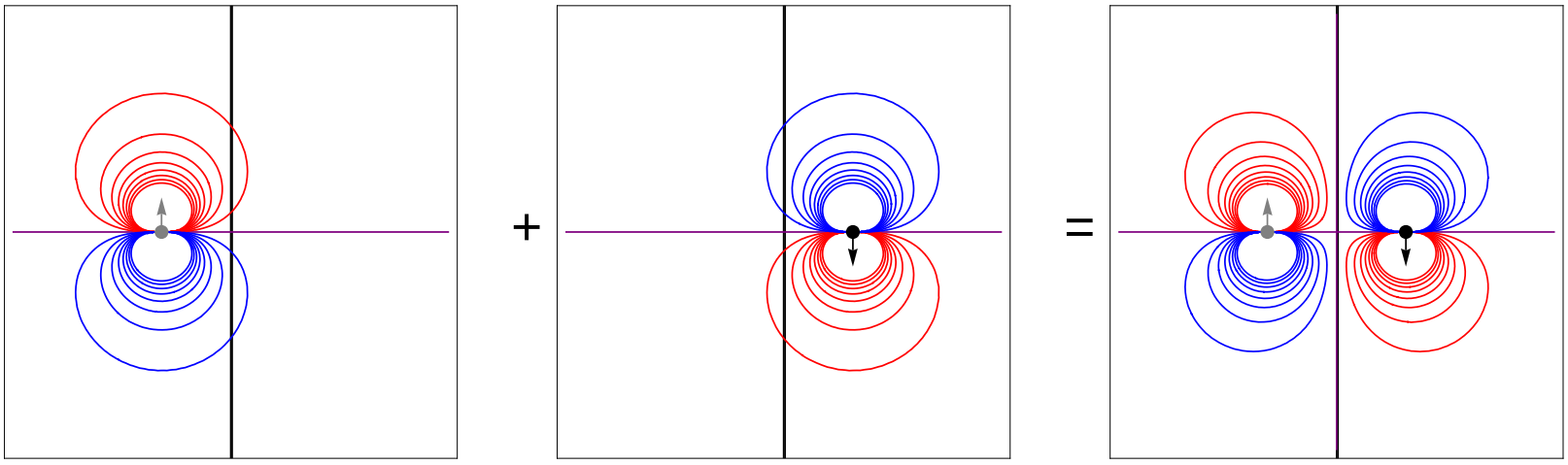


Parallel force

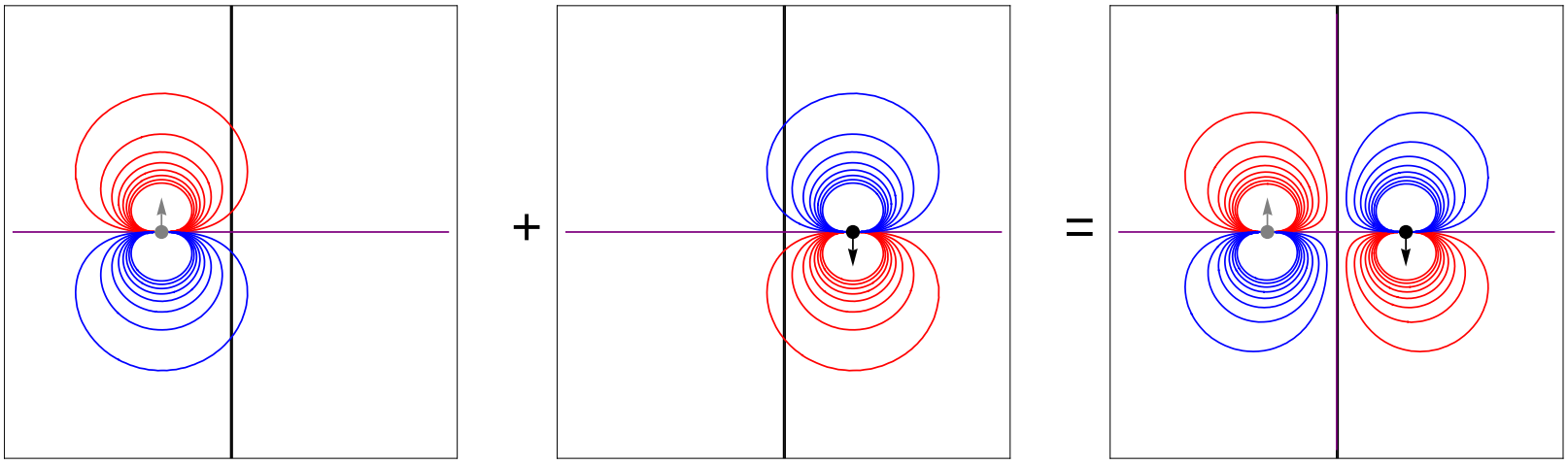


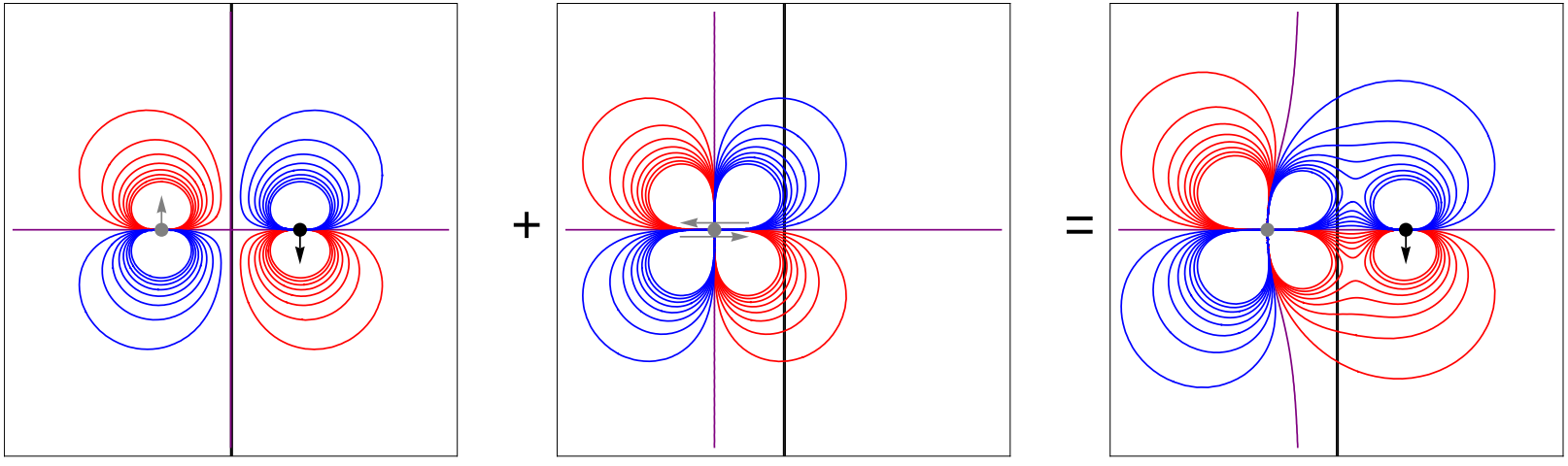
Perpendicular force

Rigid wall - the method of images

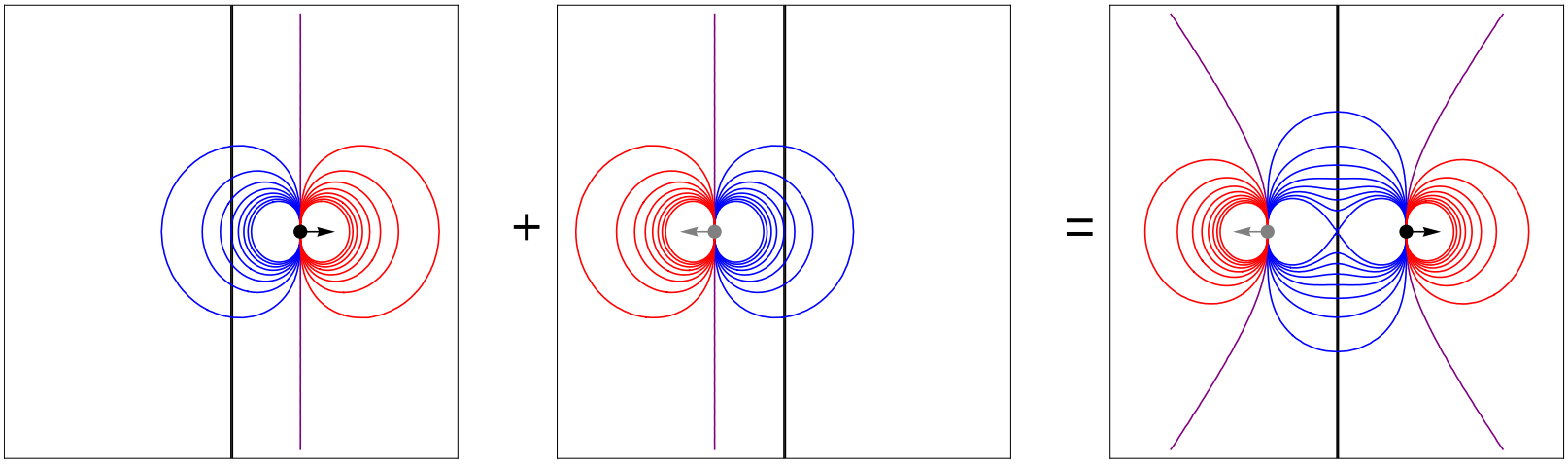


Rigid wall - the method of images

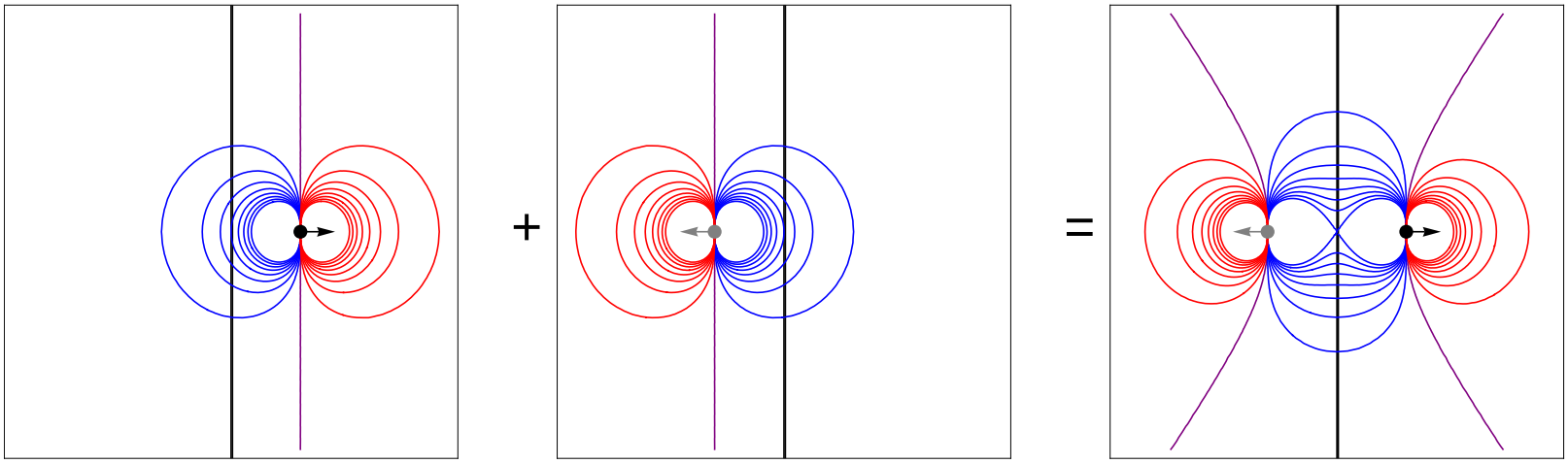


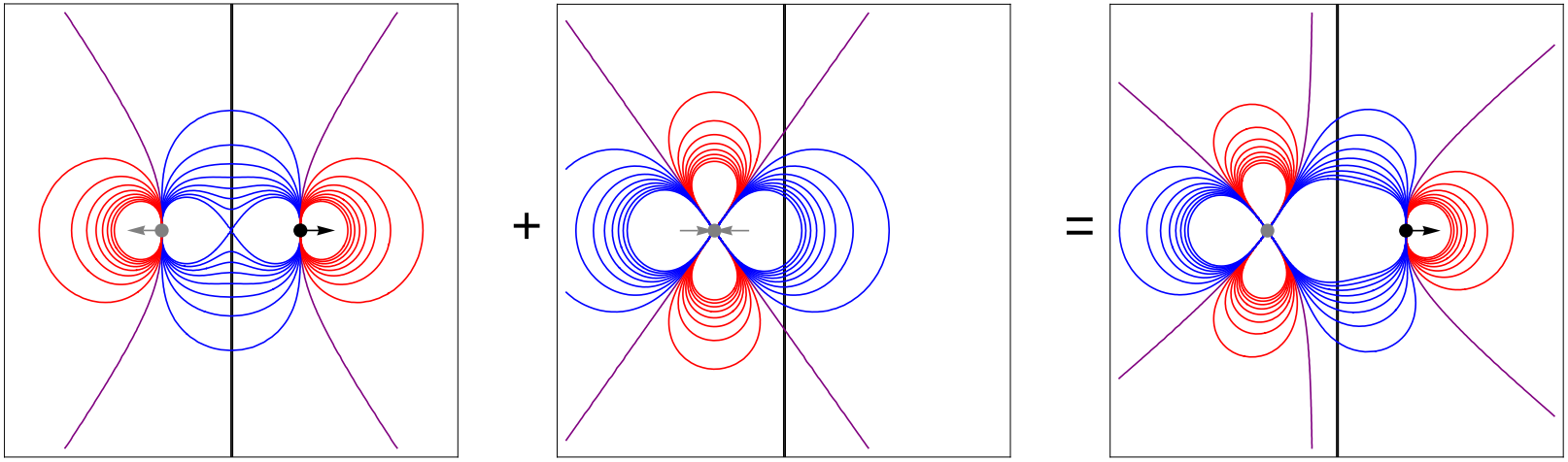


Rigid wall - the method of images



Rigid wall - the method of images





Normalisation and notation

- Units:
  - Length scale: the wall-particle distance h.
  - Pressure $p_0 = \frac{F}{4\pi h^2}$.
- The interface at $x = 0$.
- The particle near the interface in $(1, 0, 0)$.
- We calculate the pressure field at a point (x, y, z), $r = \sqrt{x^2 + y^2}$.

Normalisation and notation

- The range of coordinates in every plot

$$x \in (-3.14, 3.14) \qquad y \in (-3.14, 3.14)$$

- The isobars

$$p \in \{0, \pm 0.25, \pm 0.5, \pm 0.75, \pm 1, \pm 1.25, \pm 1.5, \pm 1.75, \pm 2\}.$$

- Colour: red for positive pressure, blue for negative pressure, purple for $p = 0$.
- The images are gray, real point particles are black.

Pressure field generated by multipoles in an unbounded fluid

- Point force $\mathbf{F} \parallel -\mathbf{e}_y$

$$p_1(x,y) = -\frac{y}{r^3}$$

- Point force $\mathbf{F} \parallel \mathbf{e}_x$

$$p_2(x,y) = \frac{x}{r^3}$$

- Stokes doublet (dipole)

$$p_{SD1}(x,y) = -\frac{3xy}{r^5} \qquad \left(\begin{array}{c}\longleftarrow\\ \bullet \\ \longrightarrow\end{array}\right)$$

- Stokes doublet (dipole)

$$p_{SD2}(x,y) = \frac{1}{r^3} - \frac{3x^2}{r^5} \qquad (\leftarrow \bullet \rightarrow)$$

Pressure field in a constrained fluid

Boundary $\parallel \mathbf{e}_y$

- Free boundary, $\mathbf{F} \parallel \mathbf{e}_y$

$$p_1(x-1,y) + p_1(x+1,y)$$

- Free boundary, $\mathbf{F} \perp \mathbf{e}_y$

$$-p_2(x+1,y) + p_2(x-1,y)$$

- Rigid wall, $\mathbf{F} \parallel \mathbf{e}_y$

$$p_1(x-1,y) - p_1(x+1,y) + 2p_{SD1}(x+1,y)$$

- Rigid wall, $\mathbf{F} \perp \mathbf{e}_y$

$$p_1(x-1,y) - p_1(x+1,y) + 2p_{SD2}(x+1,y)$$

Literature

1. J. R. Blake, *A note on the image system for a stokeslet in a no-slip boundary*, Proc. Camb. Phil. Soc., **70**, 303 (1971).
2. Kim S., Karilla S., *Microhydrodynamics. Principles and Selected Applications*, Dover, 2005.