Współrzędne cylindryczne (r, φ, z)

Gradient, dywergencja i laplasjan

$$\nabla\phi = (\frac{\partial\phi}{\partial r}, \frac{1}{r}\frac{\partial\phi}{\partial\varphi}, \frac{\partial\phi}{\partial z}) \qquad \nabla\cdot\mathbf{u} = \frac{1}{r}\frac{\partial(ru_r)}{\partial r} + \frac{1}{r}\frac{\partial u_\varphi}{\partial\varphi} + \frac{\partial u_z}{\partial z} \qquad \nabla^2\phi = \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial\phi}{\partial r}) + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\varphi^2} + \frac{\partial^2\phi}{\partial z^2}$$

Tensor naprężeń

$$\sigma_{rr} = -p + 2\eta\frac{\partial u_r}{\partial r}, \qquad \sigma_{\varphi\varphi} = -p + 2\eta\left(\frac{1}{r}\frac{\partial u_\varphi}{\partial\varphi} + \frac{u_r}{r}\right), \qquad \sigma_{zz} = -p + 2\eta\frac{\partial u_z}{\partial z},$$

$$\sigma_{\varphi z} = \eta\left(\frac{1}{r}\frac{\partial u_z}{\partial\varphi} + \frac{\partial u_\varphi}{\partial z}\right), \qquad \sigma_{rz} = \eta\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right),$$

$$\sigma_{r\varphi} = \eta\left(\frac{\partial u_\varphi}{\partial r} - \frac{u_\varphi}{r} + \frac{1}{r}\frac{\partial u_r}{\partial\varphi}\right).$$

Równanie Naviera-Stokesa

$$\frac{\partial u_r}{\partial t} + (\vec{u}\nabla)\,u_r - \frac{u_\varphi^2}{r} = -\frac{1}{\varrho}\frac{\partial p}{\partial r} + \nu\left(\triangle u_r - \frac{u_r}{r^2} - \frac{2}{r^2}\frac{\partial u_\varphi}{\partial\varphi}\right),$$

$$\frac{\partial u_\varphi}{\partial t} + (\vec{u}\nabla)\,u_\varphi + \frac{u_\varphi u_r}{r} = -\frac{1}{\varrho r}\frac{\partial p}{\partial\varphi} + \nu\left(\triangle u_\varphi - \frac{u_\varphi}{r^2} + \frac{2}{r^2}\frac{\partial u_r}{\partial\varphi}\right),$$

$$\frac{\partial u_z}{\partial t} + (\vec{u}\nabla)\,u_z = -\frac{1}{\varrho}\frac{\partial p}{\partial z} + \nu\triangle u_z.$$

Równanie ciągłości

$$\frac{1}{r}\frac{\partial(ru_r)}{\partial r} + \frac{1}{r}\frac{\partial u_\varphi}{\partial\varphi} + \frac{\partial u_z}{\partial z} = 0.$$

Współrzędne sferyczne (r, θ, φ)

Gradient, dywergencja i laplasjan

$$\nabla\phi = (\frac{\partial\phi}{\partial r}, \frac{1}{r}\frac{\partial\phi}{\partial\theta}, \frac{1}{r\sin\theta}\frac{\partial\phi}{\partial\varphi}) \qquad \nabla\cdot\mathbf{u} = \frac{1}{r^2}\frac{\partial(r^2u_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta u_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial u_\varphi}{\partial\varphi}$$

$$\nabla^2\phi = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial\phi}{\partial r}) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial\phi}{\partial\theta}) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\phi}{\partial\varphi^2}$$

Tensor naprężeń

$$\sigma_{rr} = -p + 2\eta \frac{\partial u_r}{\partial r}, \qquad \sigma_{\theta\theta} = -p + 2\eta \left(\frac{1}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r} \right), \qquad \sigma_{\varphi\varphi} = -p + 2\eta \left(\frac{1}{r \sin\theta} \frac{\partial u_\varphi}{\partial \varphi} + \frac{u_\theta}{r} ctg\theta + \frac{u_r}{r} \right),$$

$$\sigma_{\theta\varphi} = \eta \left(\frac{1}{r} \frac{\partial u_\varphi}{\partial \theta} - \frac{1}{r} u_\varphi ctg\theta + \frac{1}{r \sin\theta} \frac{\partial u_\theta}{\partial \varphi} \right),$$

$$\sigma_{r\theta} = \eta \left(\frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r} + \frac{1}{r} \frac{\partial u_r}{\partial \theta} \right), \qquad \sigma_{\varphi r} = \eta \left(\frac{1}{r \sin\theta} \frac{\partial u_r}{\partial \varphi} + \frac{\partial u_\varphi}{\partial r} - \frac{u_\varphi}{r} \right).$$

Równanie Naviera-Stokesa

$$\frac{\partial u_r}{\partial t} + (\vec{u}\nabla)\, u_r - \frac{u_\theta^2 + u_\varphi^2}{r} = -\frac{1}{\varrho} \frac{\partial p}{\partial r} + \nu \left[\triangle u_r - \frac{2u_r}{r^2} - \frac{2}{r^2 \sin\theta} \frac{\partial (u_\theta \sin\theta)}{\partial \theta} - \frac{2}{r^2 \sin\theta} \frac{\partial u_\varphi}{\partial \varphi} \right],$$

$$\frac{\partial u_\theta}{\partial t} + (\vec{u}\nabla)\, u_\theta + \frac{u_r u_\theta}{r} - \frac{u_\varphi^2 ctg\theta}{r} = -\frac{1}{\varrho r} \frac{\partial p}{\partial \theta} + \nu \left[\triangle u_\theta + \frac{2}{r^2} \frac{\partial u_r}{\partial \theta} - \frac{u_\theta}{r^2 \sin^2\theta} - \frac{2\cos\theta}{r^2 \sin^2\theta} \frac{\partial u_\varphi}{\partial \varphi} \right],$$

$$\frac{\partial u_\varphi}{\partial t} + (\vec{u}\nabla)\, u_\varphi + \frac{u_r u_\varphi}{r} - \frac{u_\varphi u_\theta ctg\theta}{r} = -\frac{1}{\varrho} \frac{\partial p}{\partial \varphi} + \nu \left[\triangle u_\varphi + \frac{2}{r^2 \sin\theta} \frac{\partial u_r}{\partial \varphi} + \frac{2\cos\theta}{r^2 \sin^2\theta} \frac{\partial u_\theta}{\partial \varphi} - \frac{u_\varphi}{r^2 \sin^2\theta} \right].$$

Równanie ciągłości

$$\frac{1}{r^2} \frac{\partial \left(r^2 u_r \right)}{\partial r} + \frac{1}{r \sin\theta} \frac{\partial (\sin\theta\, u_\theta)}{\partial \theta} + \frac{1}{r \sin\theta} \frac{\partial u_\varphi}{\partial \varphi} = 0.$$

Dla przestrzeni euklidesowej zachodzą następujące tożsamości wektorowe:

1. $rot\, rot\, \vec{v} = grad\, div\, v - \Delta \vec{v}$
2. $grad(\vec{u} \cdot \vec{v}) = (\vec{u} \cdot grad)\vec{v} + (\vec{v} \cdot grad)\vec{u} + \vec{u} \times rot\, \vec{v} + \vec{v} \times rot\, \vec{u}$
3. $rot(\vec{u} \times \vec{v}) = \vec{u}\, div\, \vec{v} + (\vec{v} \cdot grad)\vec{u} - \vec{v}\, div\, \vec{u} - (\vec{u} \cdot grad)\vec{v}$
4. $div(\vec{u} \times \vec{v}) = \vec{u} \cdot rot\, \vec{v} - \vec{v} \cdot rot\, \vec{u}$
5. $div\, rot\, \vec{u} = 0$
6. $rot\, grad\, \varphi = 0$