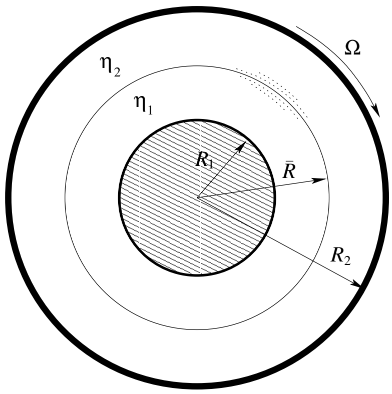
Hydrodynamics and Elasticity 2025/2026

Sheet 11

This sheet is not graded but it covers new problems and is intended to help you prepare for the exam.

Problem 1. Couette flow of two immiscible fluids Determine the velocity profile of two different immiscible fluids (with viscosities η_1 and η_2) confined between two coaxial cylinders (see figure). The radii of the cylinders are R_1 and R_2, and $\bar{R}$ is the radius of the interface between the two fluids ($R_1 < \bar{R} < R_2$). The inner cylinder is stationary, while the outer cylinder rotates about its axis with angular velocity Ω. Assume that the flow has only the azimuthal component, and that the cylindrical symmetry of the interface between the two fluids is preserved during the motion.



Problem 2. Cross-flow Viscous fluid flows between two rigid boundaries $y = 0$, $y = h$, the lower boundary moving in the x-direction with constant speed U, the upper boundary being at rest. The boundaries are porous, and the vertical velocity v_y is $-v_0$ at each one, v_0 being a given constant (so that there is an imposed flow across the system). Show that the resulting flow is

$$v_x = U\left(\frac{e^{-v_0 y/\nu} - e^{-v_0 h/\nu}}{1 - e^{-v_0 h/\nu}}\right), \qquad v_y = -v_0.$$

Sketch that the horizontal velocity profile $u(y)$ for (a) $v_0 = 0$ and (b) $v_0 h/\nu \gg 1$. Hence see that in the limit (b), the large downflow confines the vorticity to a very thin layer adjacent to $y = 0$.

Problem 3. Viscous decay of a vortex Consider a line vortex in a viscous incompressible fluid. The velocity field associated with this vortex at $t = 0$ is given by

$$v_r = 0, \qquad v_\theta = \frac{\Gamma_0}{2\pi r},$$

where Γ_0 is a constant. Show that the circulation

$$\Gamma(r,t) = 2\pi r v_\theta(r,t),$$

satisfies the evolution equation

$$\frac{\partial \Gamma}{\partial t} = \nu\left(\frac{\partial^2 \Gamma}{\partial r^2} - \frac{1}{r}\frac{\partial \Gamma}{\partial r}\right).$$

Solve this equation by seeking a similarity solution (just like the one in classes). The initial condition is $\Gamma(r,0) = \Gamma_0$ and, since we require v_θ to be finite at $r = 0$ during the evolution, we impose $\Gamma(0,t) = 0$ for $t > 0$. Find the resulting azimuthal velocity profile $v_\theta(r,t)$ and vorticity $\boldsymbol{\omega}(r,t) = \nabla \times \boldsymbol{v}$. How does the flow $v_\theta(r,t)$ look for $r \ll \sqrt{\nu t}$? Describe the evolution of vorticity.

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