

Hydrodynamics and Elasticity 2025/2026

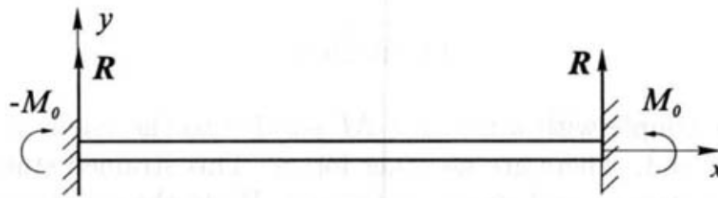
Sheet 5

One of the problems will be handed in and marked. If sending solutions over e-mail, please address them to Agnieszka.Makulska@fuw.edu.pl

Problem 1 Consider again the pipe/sausage problem analyzed during the lecture, but this time do not assume that the system is clamped in the axial direction (i.e. allow nonzero u_z). What is the deformation, strain and stress tensor in such a case?

Problem 2 A horizontal beam of length l and a circular cross section (with radius a), the ends of which have been fixed to two walls (see drawing), deforms under its own weight. The mass of the beam per unit length is q , and its Young's modulus is E .

- (a) Find the function $y(x)$ describing the shape of the deformed beam.
- (b) Find the deflection of the beam $y_{\max}$ at the point of maximal displacement.
- (c) Being fixed at both ends, the beam exerts reaction forces $\mathbf{R}$ and torques $\mathbf{M}_0$ at its ends (see drawing). Find them.



Problem 3 Consider a cylindrical pipe with inner radius a , outer radius b and infinite length made from homogeneous and isotropic elastic material. The external surface of the pipe is fixed (no deformation there). The pipe is deformed by the axial tension $\tau_0 \mathbf{e}_z$ applied uniformly along its internal surface. Find the deformation field and stress tensor in such a system.

Hint: Symmetry of the problem implies that $\mathbf{u} = u_r(r)\mathbf{e}_r + u_z(r)\mathbf{e}_z$.

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