Hydrodynamics and Elasticity 2023/2024

Sheet 2

One of the problems will be handed in and marked.

Problem 1 Consider a cylindrical rod of radius R, with its axis parallel to $\boldsymbol{e}_3$ in Cartesian coordinates. The rod is deforming according to

$$x_1 = x_1^0 - \alpha(t) x_2^0 x_3^0, \tag{1}$$

$$x_2 = x_2^0 + \alpha(t) x_1^0 x_3^0, \tag{2}$$

$$x_3 = x_3^0. \tag{3}$$

(a) Find, at time t, the position of particles which at tome $t = 0$ constituted: (i) the cross-section of the rod, at $x_3^0 = \text{const}$, (ii) a section of the cross-sectional radius, (iii) a section parallel to the cylinder axis, and located on its surface.
(b) Find the deformation field $\boldsymbol{u}$ and the strain tensor $\boldsymbol{E}$.

Interlude: Principal Stresses For any real, symmetric tensor (here: the stress tensor), there exist at least three mutually perpendicular directions (determined by its eigenvectors), which are called the principal directions. The planes perpendicular to these directions are called principal planes. On these planes, the stress vector is parallel to the normal vector (so there are no shear stresses), and the normal stresses are called principal stresses. The principal stresses (the eigenvalues of the stress tensor) include both the maximum and minimum values of the normal stresses among all possible planes passing through the given point.

Problem 2 The stress state in a medium in which the only non-vanishing components of the stress tensor are a pair of shear stresses is called *simple shear*. Consider a stress tensor for which $T_{12} = T_{21} = \tau$, and all other $T_{ij} = 0$. Find the principal directions and the principal values of the stresses in such a state.

Problem 3 For the displacement field

$$u_1 = k(2X_1 + X_2^2), \qquad u_2 = k(X_1^2 - X_2^2), \qquad u_3 = 0,$$

where $k = 10^{-4}$,
(a) Find the elongations and the change in the angle between two small line elements $d\boldsymbol{X}_1 = dX_1 \boldsymbol{e}_1$ and $d\boldsymbol{X}_2 = dX_2 \boldsymbol{e}_2$, which are attached to a particle that was initially located at $\boldsymbol{X} = \boldsymbol{e}_1 - \boldsymbol{e}_2$.
(b) Find and sketch the positions of the line elements $d\boldsymbol{X}_1$ and $d\boldsymbol{X}_2$ before and after deformation.

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