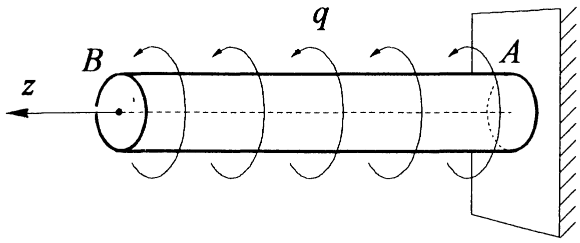
Hydrodynamics and Elasticity
Test I
13 November 2025

Each problem is worth 10 points. Please sign every sheet of paper used for your solutions.

1. Consider torques uniformly distributed over the length act on a rod of length L with circular cross section of radius R, rigidly fixed at the end A (see drawing). The torque distribution intensity (i.e., the magnitude of the torque per unit length of the rod) equals q. Find the twisting angle φ of the end B of the rod.



2. A cylider of radius R and infinite length is rotating about its axis (z) with a constant angular velocity ω. The material of the cylinder has the density ρ_0 and is characterised by Lamé constants λ and μ.

(a) Derive the formula for the divergence of the deformation field $u_r(r)$, where r is the radius in cylindrical coordinates.

(b) Find the deformation field assuming that the outer wall of the cylinder is stress-free. Find the distribution of stresses arising from the rotation.

(c) Interpret the obtained components of the strain tensor. Is the material of the cylidner stretched or compressed in the radial direction (i.e. along $\boldsymbol{e}_r$)? Is there a point at which the material is neither stretched, nor compressed?

(d) If the cylinder is rotating fast enough, the resulting stresses may lead to the material breaking. If this is the case, where can we expect a crack to appear?

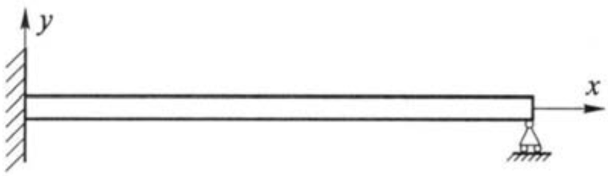
Hint In polar coordinates (r, θ), the gradient operator is

$$\boldsymbol{\nabla} = \boldsymbol{e}_r \frac{\partial}{\partial r} + \boldsymbol{e}_\theta \frac{1}{r} \frac{\partial}{\partial \theta},$$

and the curl operator for a vector field $\boldsymbol{A}$ reads

$$\nabla \times \boldsymbol{A} = \left[\frac{1}{r} \frac{\partial}{\partial r}(r A_\theta) - \frac{1}{r} \frac{\partial A_r}{\partial \theta} \right] \boldsymbol{e}_z$$

3. A horizontal beam of length L, rigidly fixed at one end and simply supported (hinged) at the other, deforms under the action of its own weight (see drawing below). Determine the deformed shape $y(x)$, and the position and the value of maximal deflection.



Good luck! *Rafał Błaszkiewicz, Maciej Lisicki & Piotr Szymczak*