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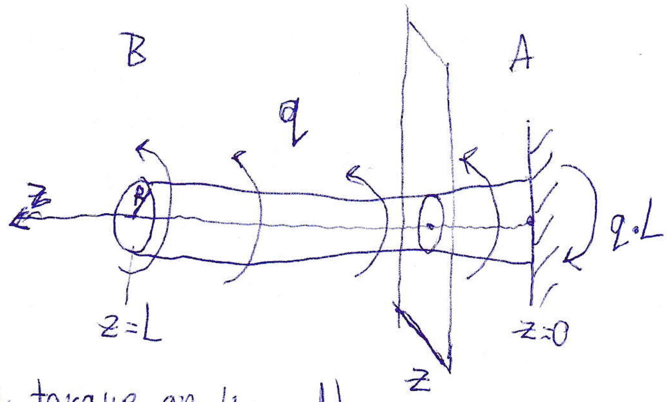
Hydrodynamics and Elasticity
Test 1

1. ~~Considering a surface at z:~~



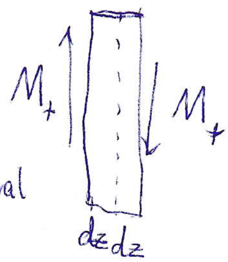
~~Total~~ For a rod to not move there must be a torque $q \cdot L$ on the wall. (counteracts total torque on the rod)

~~For any~~ For a surface at z there's a torque $q \cdot (L-z)$ on the left, and $qz - qL$ on the right.

~~Fu~~ $\tau = \frac{d\varphi}{dz}$ - twisting angle   that means: $M_t = q(L-z)$

~~From formula~~

Twisting moment: $M_t = \underbrace{\mu J}_{const} \tau$ μ - Lamé constant
J - cross section integral



$$\tau = \frac{M_t}{\mu J} = \frac{q}{\mu J}(L-z)$$

$$J = \iint (x^2+y^2)\,dx\,dy = \int_0^R \int_0^{2\pi} (r^2)\, r\,dr\,d\varphi = 2\pi \int_0^R r^3\,dr = 2\pi \frac{R^4}{4} = \frac{\pi R^4}{2}$$

$$\frac{d\varphi}{dz} = \frac{q}{\mu J}(L-z), \quad \varphi(0) = 0$$

$$\varphi(z) = \frac{q}{\mu J}\left(Lz - \frac{z^2}{2}\right) + C$$

$$\varphi(0) = 0 = \frac{q}{\mu J}(0) + C \Rightarrow C = 0$$

Twisting angle of the end B

$$\varphi(L) = \frac{q}{\mu J}\left(L^2 - \frac{L^2}{2}\right) = \frac{qL^2}{2\mu J}$$

$$\boxed{\varphi(L) = \frac{qL^2}{\mu \pi R^4}}$$

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PS