Problem 4

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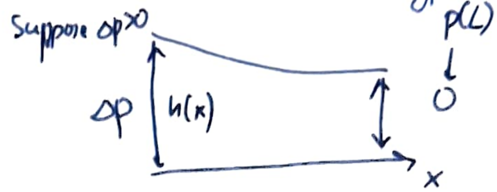

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$h(x) = h_0 + \alpha p(x)$

$$12\mu q = -\left(h_0 + \alpha p(x)\right)^3 p' =$$

$$= -\left(h_0^3 + 3h_0^2 \alpha p + O(\alpha)\right) p'$$

$$12\mu q + \left(h_0^3 + 3h_0^2 \alpha p\right) p' = 0$$

For $\alpha = 0$ $\quad 12\mu q + h_0^3 p' = 0$

$$-p' = \frac{\Delta p}{L} = \frac{12\mu q_0}{h_0^3} \qquad q_0 = \frac{\Delta p h_0^3}{12\mu L}$$

$$\int \left(1 + 3\frac{\alpha p}{h_0}\right) dp = -\int \frac{12\mu q}{h_0^3} dx$$

$$p(x) + \frac{3}{2}\frac{\alpha}{h_0} p^2(x) = -\frac{12\mu q}{h_0^3} x + C = \frac{12\mu q}{h_0^3}(L - x)$$

$p(L)$: $\quad 0 = -\frac{12\mu q}{h_0^3} L + C \qquad C = \frac{12\mu q}{h_0^3} L$

$$\Delta p + \frac{3}{2}\frac{\alpha}{h_0}\Delta p^2 = \frac{12\mu q}{h_0^3} L = \underbrace{\frac{12\mu L}{\Delta p h_0^3}}_{1/q_0} \Delta p\, q = \Delta p \frac{q}{q_0}$$

$$1 + \frac{3}{2}\frac{\alpha}{h_0}\Delta p = \frac{q}{q_0}$$

$$q = q_0\left(1 + \frac{3}{2}\frac{\alpha \Delta p}{h_0}\right)$$

Compliance increases flux, which makes sense because effectively the channel gets ~~longer~~ taller on average (wherever $p > 0$), so the pressure gradient $|\nabla p|$ is smaller on average, leading to more fluid moving through the outlet (which is still $h(L) = h_0$), due to increased pressure drop there

$p(x)$ from quadratic formula

$$\frac{3}{2}\frac{\alpha}{h_0} p^2 + p - \frac{12\mu q}{h_0^3}(L - x) = 0$$

insert solution of q

$$p(x) = \frac{1}{2 \cdot \frac{3}{2}\frac{\alpha}{h_0}}\left(-1 \pm \sqrt{1 + \frac{6\alpha}{h_0} \cdot \frac{12\mu q}{h_0^3}(L - x)}\right) = \frac{1}{3\frac{\alpha}{h_0}}\left(\sqrt{1 + \frac{6\alpha}{h_0}\Delta p\left(1 + \frac{3}{2}\frac{\alpha \Delta p}{h_0}\right)\frac{L - x}{L}} - 1\right)$$

should be $+$