Kacper Lubkowski

P2

10 pkt

Mass inside the tube is $M(t) = \rho l A(t)$, $\rho = \text{const}$ - incompressible $\Rightarrow \dot{M} = \rho l \dot{A}$

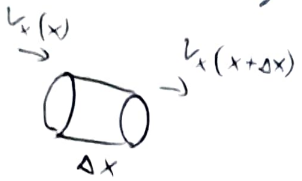
$\dot{M}$ has to be equal to the flux, so $\dot{M} = -2 \rho v_x\left(\frac{l}{2}\right) A(t)$

↑ because 2 sides

$$\Rightarrow \rho l A_0 \omega \sin(\omega t) = -2 \rho v_x\left(\tfrac{l}{2}\right) \cos(\omega t) A_0$$

$$\Rightarrow v_x\left(\tfrac{l}{2}\right) = \omega \tfrac{l}{2} \tan(\omega t) \quad (1) \quad ✓$$

Now for a general fragment of length Δx:



$$\rho \Delta x \dot{A} = \rho A(t) \left(v_x(x) - v_x(x+\Delta x)\right) \qquad x \geq 0$$

$$\Rightarrow \frac{\dot{A}}{A} = -v_x' \Rightarrow v_x = -\frac{\dot{A}}{A} x + B, \text{ but } v_x(0) = 0, \text{ so } B = 0$$

$$\Rightarrow \boxed{v_x = \omega x \tan(\omega t)}$$ - the same as (1), but from symmetry, we can see it changes sign at $x = 0$.

ok

Point [1] and [2] are on the axis, so there are no radial components.

From Euler: $\hat{e}_x$ component, in the middle only v_x survives

$$\frac{\partial v_x}{\partial t} + v_x \partial_x v_x = -\partial_x p \Rightarrow \partial_t v_x + \tfrac{1}{2}\partial_x (v_x^2) = -\partial_x p$$

integrate from 0 to $\frac{l}{2}$ over x

$$\partial_t \int_0^{\frac{l}{2}} \omega x \tan(\omega t)\, dx + \tfrac{1}{2} (\omega x \tan(\omega t))^2 \Big|_0^{\frac{l}{2}} = p(0) - p\left(\tfrac{l}{2}\right)$$

$$\omega^2 \sec^2(\omega t) \frac{l^2}{8} + \frac{\omega^2 l^2}{8} \tan^2(\omega t) = p(0) - p_0 \qquad \sec^2 x + \tan^2 x$$

✓

$$\Rightarrow \boxed{p(0) - p_0 = \frac{\rho \omega^2 l^2}{8} \left(\frac{\sin^2(\omega t) + 1}{\cos^2(\omega t)} \right)}$$