PROBLEM 1

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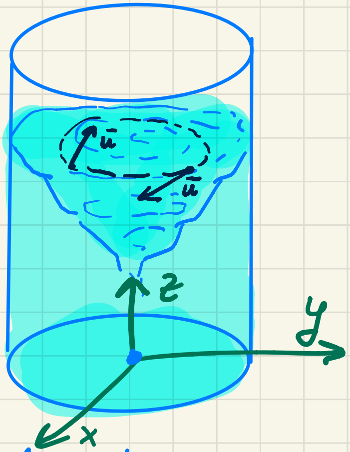

Cylindrical cross-section of rotating fluid.

Euler equation for stationary flow $(\partial_t \bar{u} = 0)$

$$(\bar{\nabla} \times \bar{u}) \times \bar{u} = -\bar{\nabla}\left(p + \psi + \tfrac{1}{2}u^2\right)$$

$$\bar{\nabla} \times \bar{u} = \begin{vmatrix} e_x & e_y & e_z \\ \partial_x & \partial_y & \partial_z \\ \Omega y & -\Omega x & 0 \end{vmatrix} = -\Omega \hat{e}_z - \Omega \hat{e}_z = -2\Omega \hat{e}_z$$

$$(\bar{\nabla} \times \bar{u}) \times \bar{u} = \begin{vmatrix} e_x & e_y & e_z \\ 0 & 0 & -2\Omega \\ \Omega y & -\Omega x & 0 \end{vmatrix} =$$

$$= -2\Omega^2 y \hat{e_y} - 2\Omega^2 x \hat{e_x} = -2\Omega^2 (x\hat{e_x} + y\hat{e_y}) = -\Omega^2 \bar{\nabla}(x^2 \hat{e_x} + y^2 \hat{e_y}) =$$

$$= -\Omega^2 \bar{\nabla}(r^2) = \bar{\nabla}\cdot(-\Omega^2 r^2)$$

$\Rightarrow$

$$(\bar{\nabla} \times \bar{u}) \times \bar{u} = -\bar{\nabla}\left(\psi + p + \tfrac{1}{2}\Omega^2 r^2\right)$$

$$-\bar{\nabla}(\Omega^2 r^2) = -\bar{\nabla}\left(\frac{p}{\rho} + gz + \tfrac{1}{2}\Omega^2 r^2\right)$$

$$\bar{\nabla}\left(\frac{p}{\rho} + gz - \tfrac{1}{2}\Omega^2 r^2\right) = 0 \Rightarrow \frac{p}{\rho} + gz - \tfrac{1}{2}\Omega^2 r^2 = \underset{\text{const.}}{C}$$

(The formula in the problem corresponded to irrotational flow, and here we have non-zero rotation)

for the free surface: $-\dfrac{p_{atm}}{\rho g} + C + \dfrac{1}{2g}\Omega^2 r^2 = z(r) \Rightarrow$

$$z(r) = C' + \frac{1}{2g}\Omega^2 r^2$$