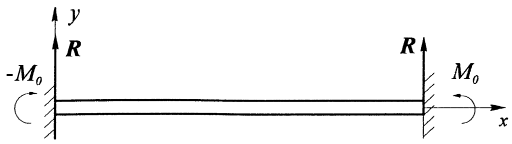
Hydrodynamics and Elasticity 2023/2024

Sheet 6

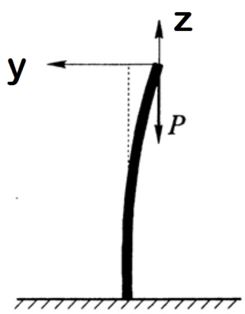
One of the problems will be handed in and marked.

Problem 1 A horizontal beam of length l and a circular cross section (with radius a), the ends of which have been fixed to two walls (see drawing), deforms under its own weight. The mass of the beam per unit length is q, and its Young's modulus is E.

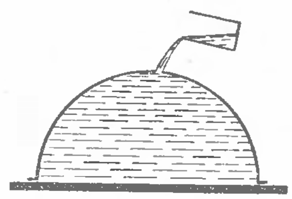
1. Find the function $y(x)$ describing the shape of the deformed beam.
2. Find the deflection of the beam y_{max} at the point of maximal displacement.
3. Being fixed at both ends, the beam exerts reaction forces R and torques M_0 at its ends (see drawing). Find them.



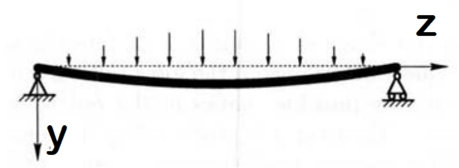
Problem 2 Analyze the buckling problem for a stick of length l rigidly fixed at one of the ends and free on the other. What is the critical load at which such a stick will buckle?



Problem 3 A hemispherical shell, which sticks to the flat surface of a table, has a small opening at the top, through which we are pouring water inside. When the water level reaches the opening (see drawing), the water lifts the shell and as the result spills out at the bottom. Find the mass of the shell if its radius is R, the density of water is ρ, and the atmospheric pressure is p_0.



(*) Problem 4 **Roof collapse problem**: A beam of length l is simply supported† and has initial small deflection $\eta_0 \sin \frac{\pi z}{l}$. The beam is subjected to a system of transverse forces distributed along the length of the beam and proportional to the value of deflection: $K = \alpha\eta$. (Such a force may be caused by rain water collecting on top of a deflected roof). Find the limiting value of the coefficient α at which a beam of a given length l remains in stable state.



† A beam is referred to as simply supported if both its ends are hinged, and one of the hinges can freely slide in the axial direction.

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