Hydrodynamics and Elasticity 2023/2024

Sheet 3

One of the problems will be handed in and marked.

Problem 1 Consider the following state of stress in a material

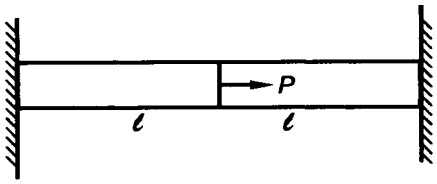
$$\widehat{\boldsymbol{T}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & T_{33}(x_1, x_2) \end{bmatrix}$$

Show that in the absence of external forces the general form of T_{33} is

$$T_{33}(x_1, x_2) = \alpha x_1 + \beta x_2 + \gamma.$$

Hint Use the structure of the components of the strain tensor $\widehat{\boldsymbol{E}}$. By definition, the components E_{ij} are not independent. They are related by the so-called compatibility conditions of the strain tensor. They also imply that not every tensor of rank 2 can be a strain tensor. First derive relationships between elements of $\widehat{\boldsymbol{E}}$ (by taking appropriate derivatives and equating them), and then use them to find the general form of T_{33}.

Problem 2 A composite rod, formed by welding two slender bars of equal length and diameter, is loaded by an axial force P as shown in the figure. If Young's moduli of the two portions are E_1 i E_2 find how the applied force is distributed between the two halves. The external walls are stiff and they do not deform.

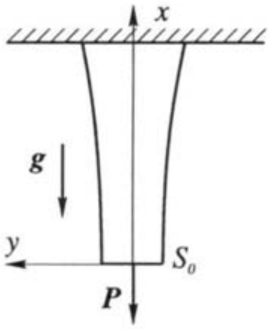


Problem 3 Consider a cylindrical rod that is acted upon by an axial stress $T_{11} = -P$. What will be the state of stress in the rod if the lateral surface is constrained so that there is no contraction or expansion? Show that the effective Young's modulus $(E_Y)_{eff} = T_{11}/E_{11}$ is equal to

$$(E_Y)_{eff} = E_Y \frac{(1-\nu)}{(1-2\nu)(1+\nu)},$$

where ν is the Poisson ratio.

(*) Problem 4 A rod the cross section of which slightly varies along its length is suspended in a vertical plane, under the action of gravity and a force P uniformly distributed over the lower cross section S_0. Of what shape should it should be (find $S(x)$) for the tensile stress T_{xx} to be identical in every cross section? Assume that T_{xx} is the only nonzero component of the stress tensor.



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